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T-duality and Gauge Symmetry in Supermembrane Theory

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ABSTRACT

T-duality has been shown to arise from a gauge symmetry in some string theories. However, in the case of type IIA superstring compactified on a circle, it is not possible to show that this is the case. This situation is rather uncomfortable since string string duality suggests that all T-dualities should arise from a gauge symmetry. Here we show that the T-duality of type IIA superstring compactified on a circle arises from the reparametrization of the supermembrane. Then we show how this reparametrization can be understood in terms of a gauge symmetry of the supermembrane, thus allowing to connect T-duality in type II superstring to a gauge symmetry of the supermembrane.

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1 Introduction

Not long ago, it was argued that some of the T-dualities in string theory can be interpreted as gauge symmetries [1, 2]. This is the case of the T-duality found in the bosonic and heterotic strings compactified on a circle. On the other hand, it does not seem possible to interpret T-duality as a gauge symmetry in all cases. For example, type IIA superstring compactified on a circle of radius R has as T-dual the type IIB superstring compactified on a circle of radius $2/R$ [1]. However, there is no enhanced gauge symmetry available at the self dual radius which is needed to interpret the T-duality as a gauge symmetry. This obstacle should be overcommed if we are going to take string string duality and string unification seriously. It would be strange to find that T-duality in heterotic string theory which can be interpreted as a gauge symmetry suddenly looses this property after a string-string duality transformation. It is not natural to have T-duality in heterotic string arise from a gauge symmetry and not see the same thing happening in the type II superstring; specially when we have evidence that both theories are phases of the same M-theory.

There are three reasons which tempt us to ask if it is possible that the T-dualities of type II superstring compactified on a circle follow from a gauge symmetry of the membrane action. First, the interaction of both type II superstring theories compactified on a circle are equivalent [1]. Second, string theory can be obtained from membrane theory after dimensionally reducing both the world volume and spacetime [3]. Third, membrane theory is conjectured to be non renormalizable because it contains both perturbative and nonperturbative effects of string theory [4][2]; thus, all string interactions are included in the membrane action. The natural setting to answer this question was provided in [8] where it was shown, examining the non perturbative spectrum, that type IIB superstring and type IIA superstring, both compactified on a circle should be identified with 11 dimensional supergravity compactified on a

[2]This was also argued in [7] where it was shown that type IIA superstring in the strong coupling limit yields 11-dimensional supergravity.

torus. Since 11 dimensional supergravity is believed to be the low energy limit of membrane theory [9, 11] we should consider the membrane compactified on a torus to show that the T-duality of type II superstring compactified on a circle arises from a reparametrization invariance of the membrane which can be shown to follow from a gauge symmetry present in the supermembrane action. This result provides more evidence on the conjecture that the different string theories are different phases of the supermembrane. It also provides a further step in the unification of T-dualities which is essential in the understanding of string-string duality.

The strategy used in [1] to show that T-duality in the heterotic string is a gauge symmetry consists in showing that at the enhanced symmetry point the action of T-duality on the vertex operator of the gauge field can be interpreted as a rotation of the SU(2) gauge group about the “x-axis” of the SU(2), and is therefore a gauge symmetry. Despite the demonstration taking place at the self dual radius, the duality transformation is a gauge transformation everywhere in the moduli space. In the type II superstring there are no points in the moduli space at which the theory acquires additional gauge symmetry, and therefore, it is not possible to show that T-duality is a gauge symmetry of this theory.

We shall show that T-duality in type II superstring is a reparametrization of the supermembrane. We will then consider a particular point in the moduli space of the membrane where the U(1) symmetry is enhanced to an SU(2) symmetry. This reparametrization of the membrane at the enhanced symmetry point, which is interpreted as a T-duality transformation in string theory, may then be interpreted as a spacetime gauge symmetry: it can be treated as an SU(2) rotation by π about the “x-axis”. It would be interesting to see how our work is related to that of [16] which also deals with the interpretation of T-dualities in M-theory.

The action for the membrane [9] is given by

$$S = \int d^3\zeta(\frac{1}{2}\sqrt{-g}g^{ij}E_i^a E_j^b \eta_{ab} - \frac{1}{6}\epsilon^{ijk}E_i^A E_j^B E_k^C A_{CBA} - \frac{1}{2}\sqrt{-g}). \tag{1}$$

Here, E_i^A is a supervielbein and A_{CBA} is a super three-form. The world volume

has coordinates ζ and metric g_{ij}. Among the symmetries of this action there is local reparametrization symmetry. The presence of a fermionic κ symmetry and supersymmetry will allow us to concentrate on the bosonic sector. The bosonic sector of action (1) is given by

$$S = \int d^3\zeta (\frac{1}{2}\sqrt{-g}g^{ij}\partial_i x^m \partial_j x^n G_{mn} - \frac{1}{6}\epsilon^{ijk}\partial_i x^m \partial_j x^n \partial_k x^p A_{pnm} - \frac{1}{2}\sqrt{-g}). \qquad (2)$$

The x^m, $m = 1, ..., 11$ are coordinates on an 11 dimensional manifold with metric G_{mn}. As suggested in [8], the supermembrane compactified on a torus should be equivalent to type II superstring string compactified on a circle which suggest that two of the coordinates should be coordinates on a torus and thus have periodic boundary conditions

$$x^I = x^I + 2\pi R^I, \quad I = 10, 11 \qquad (3)$$

The moduli space of the torus is characterized by complex and Kahler structure deformations. It is then possible to pick a point in the moduli space of the torus such that

$$R^{11}R^{10} = 2. \qquad (4)$$

In order to obtain a string theory from action (1) we must dimensionally reduce the action in the same way as was done in [3]. First we split the world volume coordinates

$$\begin{aligned} \zeta^i &= z^i \quad i = 1, 2 \\ \zeta^3 &= \rho. \end{aligned} \qquad (5)$$

We then may use the reparametrization invariance to set

$$\rho = x^{10} \qquad (6)$$

or to set

$$\rho = x^{11} \qquad (7)$$

Standard techniques in dimensional reduction [15] suggest we write the world volume metric in the form

$$g^{ij} = \phi^{-2/3} \begin{pmatrix} \hat{g}^{ij} + \phi V_i V_j & \phi^2 V_i \\ \phi^2 V_j & \phi^2 \end{pmatrix}. \tag{8}$$

The fields ϕ and V are non dynamical. Making use of the equations of motion and the choice (6) we obtain a string action of the form

$$S = \int d^2 z \sqrt{-\hat{g}} \hat{g}^{ij} \partial_i \hat{x}^m \partial_j \hat{x}^n \hat{G}_{mn} + \frac{1}{2} \epsilon^{ij} \partial_i \hat{x}^m \partial_j \hat{x}^n \hat{A}_{nm}, \quad i,j = 1,2. \tag{9}$$

The $\hat{x}^m$, $m = 1, ..., 10$ are now coordinates on an 10 dimensional manifold with metric $\hat{G}_{mn}$. The action (9) is the Green-Schwarz action [14] for the type II superstring propagating on a circle of radius R^{10}. If we use the reparametrization invariance (7) we obtain the same action but the string now propagates on a circle of radius $2/R^{10}$. Thus, the T-duality found in type II superstring on a circle is just a certain reparametrization of the supermembrane. This result was obtained at a particular point on the moduli space of the membrane theory. Similar arguments to those used in [1] allow us to extend our result to all the points in the moduli space. Thus, this reparametrization of the supermembrane compactified on a torus is responsible for the T-duality found in the type II superstring compactified on a circle.

The fact that a reparametrization of the membrane is responsible for T-duality in the type II string should not come as a surprise. We know of other T-dualities in nine dimensions where the heterotic string is related to the type I string [10]. It turns out that this T-duality can be interpreted as a reparametrization of the open membrane which have current algebras defined at its boundaries [11]. As the open membrane which has the topology of the cylinder reduces its length, it becomes a heterotic string. As the open membrane reduces its radius, it becomes an open string, and the current algebra generates the Chan Paton factors required to define the type IA string.

We have succeeded in relating T-duality in type II superstring to a certain reparametrization invariance of the supermembrane. Now, we must relate this certain reparametrization invariance to a gauge symmetry. To do this we first note that for

a compactification of the supermembrane on a manifold which has n 2-cycles, the theory will have n gauge fields [12]. To see this it is sufficient to consider the term

$$-\int d^3\zeta \frac{1}{6}\epsilon^{ijk}\partial_i x^m \partial_j x^n \partial_k x^p A_{pnm} \tag{10}$$

in action (2). As it is done in string theory [13], we may express

$$A_{11\ 10m} = A^I_{11\ 10} A^I_m \tag{11}$$

where I labels a particular 2-cocycle (for the torus I=1), and m labels a spacetime coordinate while 10 and 11 label compact coordinates. In order argue for the existence of nonperturbative states which will enhance this U(1) symmetry to an SU(2) symmetry it is best to digress first to a K3 compactification of the supermembrane [12].

As it was explained in [5, 6], in order to have D=7 membrane/string duality, it is necessary to consider the nonperturbative states (solitonic membranes) which can wrap about the 2-cycles of K3. As a 2-cycle of K3 becomes massless it will enhance the U(1) symmetry associated to that 2-cycle to an SU(2) symmetry. The U(1) gauge fields in the supermembrane are constructed from the 3-form tensor $A_{abm} = A^I_{ab} A^I_m$ where I denotes a particular K3 two-cocycle and a, b label compact coordinates while m denotes spacetime coordinates. The states which enhance the U(1) symmetries are given by the nonperturbative states whose masses are proportional to the area of the 2-cycles.

Membrane/string duality requires this enhancement of symmetry to hold for all of the 22 cycles which vanish at particular points in the moduli space. In particular, this means that it must also hold for any of the 22 cycles of T^4/Z_2, one of which is the 2-cycle of a T^2 which is left invariant under the action of the orbifold. Thus, if the two-cocycle $A_{10\ 11}$ in (11) which is dual to the 2-cycle of T^2 vanishes, two charged states with respect to the U(1) field will become massless and enhance the symmetry to an SU(2) gauge group. The symmetry enhancement is independent of how the size of the 2-cycle vanishes. In particular, we can consider the case in which the

parallelogram describing the two-cycle reduces to a line. This is achieved by reducing either x^{10} or x^{11} to zero size. Thus, as x^{10} or x^{11} vanishes to zero size, the gauge symmetry is enhanced to SU(2).

Now consider the case in which we set

$$\rho = x^{11} \tag{12}$$

Then (10) reduces to

$$-\int d^3\zeta \frac{1}{6}\epsilon^{ij}\partial_i x^m \partial_j x^{10} A_{1011m} \tag{13}$$

where $i, j = 1, 2$. We now use the reparametrization invariance to set

$$\rho = x^{10} \tag{14}$$

Then (10) reduces to

$$-\int d^3\zeta \frac{1}{6}\epsilon^{ij}\partial_i x^m \partial_j x^{11}(-A_{1011m}) \tag{15}$$

Thus, the reparametrization that takes (12) to (14) has the effect of interchanging x^{11} with x^{10} and therefore of taking

$$A^I_m \rightarrow -A^I_m \tag{16}$$

which is also what it is expected from a T-duality transformation in string theory. In performing this reparametrization when either x^{10} or x^{11} is very small, we are actually enforcing (16) at an enhanced symmetry point. Therefore, the reparametrization invariance which takes (12) to (14) is a gauge symmetry because it can be understood as a rotation about the SU(2) “x-axis” by π, just like T-duality is understood as a gauge symmetry in the bosonic and heterotic strings [1].

This result can be extended to type II superstring theories compactified in more dimensions, for example, when compactifying type II superstring theories on a torus. In this case we may classify theories according to the radii of compactification and to whether they are type IIA superstring or type IIB superstring. T-duality in this case

can act on either of the radii of the torus. Because T-duality changes the sign of the fermionic partners of the compactified dimensions, an odd number of T-duality transformations will change the sing of Γ^{11} and thus alter the GSO projection [1]. Thus, under an odd number of duality transformations the type IIA superstring is mapped to the type IIB superstring and vice versa. On the other hand, an even number of T-duality transformations maps the type IIA superstring (type IIB superstring) to itself. In all cases, the radius on which T-duality acts is mapped to its dual. Denoting $\mathcal{D}_L$ the T-duality transformation we have the following maps

$$\begin{aligned} \mathcal{D}_{L1}|IIA, R_1, R_2> &= |IIB, 2/R_1, R_2> && (17) \\ \mathcal{D}_{L2}|IIA, R_1, R_2> &= |IIB, R_1, 2/R_2> && (18) \\ \mathcal{D}_{L1}\mathcal{D}_{L2}|IIA, R_1, R_2> &= |IIA, 2/R_1, 2/R_2> && (19) \\ \mathcal{D}_{L1}\mathcal{D}_{L2}|IIB, R_1, R_2> &= |IIB, 2/R_1, 2/R_2> && (20) \end{aligned}$$

The first two maps relate three different theories which should be identified because they have similar spectra and interactions [1]. On the other hand, the last two maps must be used to identify same theories. These maps are not gauge symmetries of the type II superstring theories. However, the T-duality maps of the type II superstring are a consequence of reparametrizations of the supermembrane. These reparametrizations of the supermembrane can take place at the points in the moduli space where one of the three 2-cycles of T^3 vanishes to enhance one of the three U(1) gauge symmetries to an SU(2) gauge symmetry. Thus, these reparametrizations can also be identified with gauge transformations. Let us denote the radii of the three-torus $R^{11},\ R^{10},\ R^9$. We shall choose the radii to satisfy

$$R^{11} = \frac{2}{R^{10}} = \frac{2}{R^9}. \qquad (21)$$

Using the reparametrization invariance of the supermembrane to set

$$\rho = x^9 \qquad (22)$$

or

$$\rho = x^{10} \tag{23}$$

or

$$\rho = x^{11} \tag{24}$$

we obtain the type II superstring theories

$$|IIA, R^{10}, R^{11} >, \tag{25}$$

$$|IIB, 2/R^{10}, R^{11} >, \tag{26}$$

$$|IIB, R^{10}, 2/R^{11} >, \tag{27}$$

respectively. Here we have adopted the convention that the gauge choice (22) yields the $|IIA, R^{10}, R^{11} >$. Thus, once more, the T-dualities which do not seem to be a symmetry of the type II superstring happen to follow from the reparametrization of the supermembrane. These reparametrizations can be understood as gauge symmetries, just like in the case of the T^2 compactification of the supermembrane.

One of the problems we have encountered in the past is the existence of certain T-dualities which arise from gauge symmetries and other T-dualities which do not. This is an obstacle to our understanding of string unification. However, it is believed that string theory is a phase of supermembrane theory and the proof that the T-dualities which do not seem to be a consequence of gauge symmetry in string theory, arise in fact from the gauge symmetry of the membrane add more evidence to the conjecture that membrane theory will unify all string theories.

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